**2008 IMSA Junior High Math Competition**

7th Grade Team Contest

1. Theresa has 12 identical pieces of candy to distribute among 10 children. How many ways can she distribute the candy, if each child must get at least one piece?
2. Allister asks an old fortune-teller for her age. She replies, “Your age is the 9th prime number. Twelve years ago, I was five times as old as you were.” How old is the fortune-teller? (Hint: 1 is not a prime number.)
3. Voltaire can write three obituaries, five rants, and two melancholy pamphlets each day. A pamphlet takes five times as long as an obituary to write, and a rant can be written in half the time of a pamphlet. If Voltaire decides to spend an entire day writing rants, how many could he write? Round *down* to the nearest rant.
4. How many distinct 5-digit numbers can be created from the digits 0, 1, 2, 3, and 4 if no digit is repeated?
5. A book is 999 pages. How many times did the digit 1 appear when numbering the pages of the book?
6. What is the sum of the first thirty odd positive integers?
7. In a basket there are 60 pieces of candy, where 15 are red, 20 are green, and 25 are blue. The basket is taken into a dark room so that the colors cannot be distinguished and candies are drawn from the basket without replacement. How many must be drawn in order to guarantee that there will be at least one of each color?
8. How many positive integral factors does 2008 have?
9. How many digits does  have?
10. Larry is making cakes for a party. The recipe calls for 2 eggs and will serve 6 people. There will be 273 people at the party. Eggs are sold by the dozen, and each dozen costs $0.89. How much money will he need to buy enough eggs to make all the cakes? Express answer in dollars and cents.
11. How many degrees are there in the exterior angle of a thirty-five-sided regular polygon? Round to the nearest tenth of a degree.
12. A map has a scale of 1.5 inches to 2 miles. If two houses have a distance of 1 yard on the map, find their actual distance in miles.
13. Two *identical* fair 6-sided dice are rolled at the same time. What is the probability that the value on one die is greater than the value on the other die?
14. Sammy sells his lawn chair to Ann for 20% more than he paid for it. Ann puts it on sale for 10% more than she paid for it. It doesn’t sell because it’s winter, so she puts it in the clearance section at 50% off her original asking price. If the final price was $66.00, how much did Sammy pay? Give your answer in dollars.
15. The probability of getting three heads after flipping a fair coin three times is *n*. The probability of getting three heads after flipping three different fair coins at the same time is *p*. Find *np*.
16. How many positive integers less than 2008 are multiples of 4 and/or 5 but not 8?
17. If  and *a, b, c, d* and *e* are positive integers such that no pair has a common factor larger than 1, find the ordered set (*a, b, c, d, e*).
18. When Hillary makes a speech, she angers the audience 30% of the time, convinces them 60% of the time, and bores them 40% of the time. She cannot do all three at the same time, neither can she do nothing to the audience. In addition, she never angers and convinces them at the same time. She both bores and convinces them 10% of the time. She angers and bores them 20% of the time. How often does she anger them without convincing or boring them? Express your answer as a percentage.
19. Two numbers have a sum of 616. The first number has 0 at its ones digit. If you divide the first number by 10, then it’s the same as the second number. What’s the difference between the two numbers?
20. A decreasing number is a number such that each digit is less than the one to its left. 2, 21, 430 and 921 are decreasing numbers. 15, 544 and 914 are not decreasing numbers. How many decreasing numbers are there?

**2009 IMSA Junior High Math Competition**

7th Grade Team Contest

1. What is the value of ?
2. Nathan draws a triangle with the following coordinates: (5,3), (2,1), (7,1) on the blackboard. Find its area!

1. Lisa has enlarged a 3 by 5 picture so that both the length and the width are tripled. By what number must the original area be multiplied to get the enlarged area?
2. Molly rolls a pair of dice once. What is the probability that the sum of the numbers rolled will be 10 or less? Express your answer as a fraction in lowest terms.
3. In 1963, mathematicians at the University of Illinois used a computer to show that  is a prime number. (Cool, huh?) If this number were written in standard notation, it would contain 3,376 digits. What is the units digit?
4. Given a list of 5 distinct nonnegative integers with a mean of 18 and a median of 19, what is the greatest possible range?
5. Each of three marbles in Sam’s bag, A, B, and C, is colored one of three colors. One marble is white, one marble is red, and one marble is blue. Exactly one of the following statements is true: (a) A is red. (b) B is not blue. (c) C is not red. What color is marble B?
6. Michel (named for Michelangelo!) finished painting 2/3 of a room by 2:00 PM and 3/4 of the same room by 2:30 PM. At this rate, when does he finish painting the room?
7. How many ordered integral triples (a, b, c) have the property that each number is the product of the other two?
8. From 1:08 PM to 5:46 PM, what is the degree measure of the angle through which the hour hand of a clock moves?
9. Given the proportions a/b=3/7 and c/b=9/14, what is the value of c/a? Express your answer as a common fraction.
10. The four interior angles of Amanda’s favorite quadrilateral are in the ratio of 2:4:4:5. In degrees, what is the measure of the smallest interior angle of the quadrilateral?

1. Mrs. Harte is grading her 7th-grade math class’s quizzes but seems to have lost the answer key to the ten True/False questions. She knows that:
   1. there are the same number of true answers as false answers
   2. the answers to questions 5 and 9 are false while question 8 is true
   3. the first and last answers are the same
   4. there are more true answers than false answers in the first five questions
   5. there are at least three false answers in a row

How many possible answer keys meet these criteria?

1. Out of 200 fish in Bella’s aquarium, 99% are guppies. How many guppies must be removed so that the percent of guppies in the aquarium is 98%?
2. The product of two positive numbers is 24, and the sum of their squares is 73. What is the square of their difference?
3. From a regular hexagon, three vertices are selected at random. What is the probability that these three vertices form an equilateral triangle or an isosceles triangle? Express your answer as a common fraction.
4. An isosceles trapezoid has bases of 10 units and 20 units and a height of 12 units. What is the perimeter of the trapezoid? An isosceles trapezoid is a trapezoid such that the non-parallel sides have equal length.
5. Which five digit number (or numbers) can you multiply by 4 to get the same number with its digits in reversed order?
6. Suppose that the operation @ is defined as . What is the value of ?
7. A number consists of three distinct digits. There are six different three digit numbers that can be formed with these three digits. If these six numbers sum up to 5106, find the three digits.

**2010 IMSA Junior High Math Competition**

7th Grade Team Contest

1. How many different ways are there to choose one president, two vice presidents, and four secretaries out of 10 distinct applicants?
2. Today is Sara’s birthday. Lucy, Sara’s roommate, needs to decorate the room before Sara comes back at 5PM from band practice; however, Lucy sleeps in and only starts decorating at 2:30PM. Because she can only decorate one-tenth of the room in half an hour, she won’t be able to finish in time. In order to make Sara’s birthday surprise perfect, Lucy hires Andrew to help her decorate, but Andrew can only decorate three-twentieths of the room in half an hour. If they work together, how many spare minutes will they have between the time they finish and 5PM?

1. If Katie has five different blouses, ten different skirts, and four different pairs of shoes, how many different outfits (consisting of one blouse, one skirt, and one pair of shoes) can she create?
2. Arjun can run around a 120m track in 120 seconds, and Charles can run around the same track in 100 seconds. If they start running at the same time from the same position on the track, how many seconds will pass before they are running alongside one another again?
3. Nirali has a bunch of dimes and pennies on her desk. If those 24 coins are worth $0.87, how many dimes does Nirali have?
4. Grace wants to paint the ceiling of a 4ft. by 7ft. room, but she only has $85. If Nolan and Webster demand $2 for every square foot that they paint, how much money will Grace have left over? Record your answer in dollars.
5. Fermi wants to get the attention of this girl he likes, but she only likes guys who are good at math. In order to impress her, he needs to buy her a box of chocolates on the *n*-th hour of 2010, assuming that 12AM-1AM on January 1st is the first hour and where *n* isthe only three-digit prime factor of 999,991. On what day should Fermi buy his gift? Express your answer in as a date, such as January 1, 2010.
6. There are 6 more girls than boys in Dr. Condie’s class of 24 students. What is the ratio of girls to boys?
7. In the cryptarithm MEMO + FROM = HOMER, every letter stands for a distinct digit between 0 and 9. What is the value of M?
8. At the carnival, three friends win three identical green balls. However, Liana won two while Irene only won one (Grace didn’t win any). If Jayanshu steals all of the balls and gives them back randomly, what is the probability that each girl keeps the same number of balls? Express your answer as a fraction reduced to lowest terms.
9. If Priya has 40 identical sugar cubes and 37 different cups of coffee, how many ways can she put sugar cubes into her coffee? (Every cup of coffee must have some sugar.)
10. Alex has a glass cube with a side length of 4 inches. If he cuts a 2-inch by 2-inch square hole into the center of each face (where the edges of each cut are parallel to the edges of the cube and each cut goes all the way through the cube), what is the volume of the remaining solid?
11. Ben and James are playing an abridged version of tennis where the “must win by two” rule is taken out. (Recall that in tennis a player must win four points - 15, 30, 40, GAME - to win a game.) If Ben has a 2/3 probability of winning a point (because his racket is better) and James has a 1/3 probability of winning a point, what is the chance that James wins the game?
12. The area of a circle is . What is the circumference of that same circle?
13. A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and x+32 are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?
14. IMSA has 185 seniors, each of whom plays on at least one of the school’s three varsity sports teams: swimming, tennis, and scholastic bowl. It so happens that 60 are on the swimming team; 124 are on the tennis team; 40 are on both the swimming and scholastic bowl teams; 32 are on both the swimming and tennis teams; and 29 are on both the tennis and scholastic bowl teams. Compute the number of seniors playing all three sports, given that twice this number are members of the scholastic bowl team.
15. If Adam pays $3 as a 15% tip on his order of pizza, how much was the original bill?
16. On an exotic island called Pr01, there are two clans - the Korenos and the Chinos. The Korenos always tell the truth, and the Chinos always lie. There is also a band of outcasts on the island full of natives that tell the truth half of the time and lie the other half of the time. One day, Jesse stumbles on this island and wanders into a camp composed of two Korenos, two Chinos, and one outcast. They make the following statements:

LAWRENCE: I don’t like Eric because he isn’t a Koreno. Also, I don’t like Karna either because he’s an outcast.

BEN: Both David and Eric are Korenos. Lawrence is a Chino.

DAVID: Oh, man, no one likes Ben because he’s a Chino. However, I like being around Lawrence, who happens to be a Koreno.

KARNA: Lawrence can dance well, so he must be a Koreno. I’m also a Koreno, so I love to hang out with him.

ERIC: I don’t like Lawrence because he isn’t a Koreno. Also, I don’t like Ben either because he’s an outcast.

Who is the outcast?

1. Dr. Fogel wants to make a batch of his legendary Fogel Fudge, but he doesn’t have any marshmallows. A pack of seven marshmallows at the store costs $1.50. If he’s planning to make 196 servings of fudge and every four servings requires three marshmallows, how much money will he spend buying marshmallows? Express your answer in dollars.
2. Jayanshu is trying to get the pack rat avatar on Neopets, and in order to get it, she needs to collect 1000 different items. If she collects 80 items on the first day, 79 on the second day, 78 on the third day, and so on and so forth, on what day will she get the avatar? Express your answer in the form “DAY ##”.

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**2011 IMSA Junior High Math Competition**

1. Michelle’s 8th grade class has 190 students. If there are 6 more boys than there are girls in her class. How many boys are there in her class?
2. What is the area of a triangle with coordinates (5,4), (5,7) and (9,4)?
3. Alison takes a number adds 500 to it, divides by 3, subtracts 86, and ends up with the same number that she started with. What is that number?
4. Haley is running clockwise around an 8 mile long track at 7 miles per hour. Her archenemy Sarah is running counterclockwise around the same track at 6 miles per hour. If they both start at the same time, how many minutes will it take until they meet? Round your answer to the nearest minute.
5. How many degrees are in the supplement to the interior angle of a regular nine-sided polygon?
6. What is the least common multiple of 52 and 117?
7. The area of trapezoid A is 35. If trapezoid A contains a base of 10 and a height of 5, what is the length of the other base?
8. What has a greater area: a circle with radius 2 or a square with perimeter 8? Please write Square or Circle as your answer.
9. What is the surface area of a cube with main diagonal 15 meters in length? Give your answer in square meters.
10. In a standard deck of 52 cards, what is the probability of drawing a red six, replacing it, and then drawing a face card (jack, queen, or king)?
11. If , then what is ?
12. Emma has older twin brothers. The product of their three ages is 147. What is the sum of their three ages?
13. Dr. Krouse can walk up stairs either one or two steps at a time. Her stepping sequence is not necessarily regular. For example, she might step up two steps, then two again, then one step, then one, then two, and then one more to climb up 9 steps. In how many different ways can Dr. Krouse walk up a 12-step stairwell?
14. Within the mathletes at JHMC, 170 students were surveyed on what type of math questions they liked: Probability, Geometry, and Number Theory. The results are listed below:

|  |  |
| --- | --- |
| Probability | 107 |
| Geometry | 90 |
| Number Theory | 71 |
| Probability and Geometry | 56 |
| Geometry and Number Theory | 40 |
| Probability and Number Theory | 47 |
| Probability, Geometry, and Number Theory | 36 |

How many students do not like questions on any of the three topics?

1. The scale of a map is 1/16 inch=5 miles. On the map, Chicago and San Francisco are 37.5 inches apart, Chicago and Nashville are 8.75 inches apart, and Nashville and San Francisco are 43.75 inches apart. What is the perimeter, in miles, of the triangle formed by these three cities?
2. How many integers satisfy the following inequality?
3. There are a total of 3993 digits used to number a book. How many pages are in the book?
4. How many positive 7-digit odd integers are palindromes? A palindrome is the same when written left to right as it is right to left, for example, 2315132.
5. How many six digit numbers have their digits increasing order? For example, 135679.
6. Let S be equal to the sum: . Find the remainder when S is divided by 1000.

2012 IMSA Junior High Mathematics Competition

1. How many sides does a heptagon have?
2. Michael, Jacob, and Bryan each choose a number. The product of Michael and Jacobs’ numbers was 24, the product of Michael and Bryans’ numbers was 30, and the product of Jacob and Bryans’ numbers was 20. What the product of all three numbers?
3. How many ways can we choose 4 elements {1, 2,…12} so that *exactly one* pair of chosen numbers is consecutive?
4. John and Mary are playing a lying game. Each round, each person will make **one statement**: it will be either true or false.

|  |  |  |
| --- | --- | --- |
| **Round** | **John** | **Mary** |
| 1 | Mary will only tell the truth. | My statement 3 is true. |
| 2 | Exactly one of the three statements I make is true. | Exactly 3 statements total will be false. |
| 3 | Mary’s statement 1 was false. | Exactly one statement this **round** is true. |

How many false statements does John tell?

1. We use  to represent the product of all values of such that *k* is an integer between l and *n*, inclusive. For example, . What is the value of ?
2. Let point *O* be on side of triangle *ABC*such that . If angle ∠ABC and ∠ACB are complementary, and , what is the area of triangle, *OAC*?
3. There is a 25% chance that it will rain on Saturday and, if it does, then there is a 20% chance that it will also rain Sunday. If there is no rain on Saturday, there is a 2/3 chance that it will rain on Sunday. It is Sunday and it is raining. What is the chance that it rained yesterday?
4. You have a quarter, nickel, and dime each made in a different year. The quarter is older than the nickel. What is the probability that the newest coin is the nickel?
5. Find the area of a circle whose circumference is 2.
6. Set *P* contains all perfect squares with less than 4 digits. Set *Q* contains all positive perfect cubes with less than 4 digits. What is the sum of all elements in *P* *Q*?
7. Jose has 15 blue cards and *k* green cards. If he draws two cards at random without replacement, the probability that the two cards are the same color is equal to the probability that they are different colors. Find all possible values for the number of green cards that Jose has?
8. How many circles of radius 1 may be placed without overlap within a rectangle of dimensions ?
9. Each of the digits 1-4 is used once to create a positive, four-digit integer. What is the probability that the integer formed is divisible by 11? Express your answer as a common fraction in simplest form.
10. How many integers between 1000 and 2012 have all three of the numbers 20, 25, and 30 as factors?
11. In a clock with the hands pointing to 4:40 PM, what is the measure, in degrees, of the acute angle between the two hands of the clock?
12. If 25 square blocks are arranged to make a , how many different combinations of 3 blocks can be selected so that no two are in the same row or column?
13. Some mathletes at the JHMC are having a car wash to raise money for to attend more math competitions. Initially, 40% of the group are girls. After two girls leave and two boys arrive, 30% of the group are girls. How many girls were initially in the group?
14. Dr. Condie’s age is X years; X is also the sum of the ages of his three children. His age Y years ago was three times the sum of their ages then. What is X/Y?
15. If , what is the largest possible value of ?
16. Consider ordered pairs, (*a, b*) of positive integers satisfying *ab* = 100. Find the minimum value of